Math 630-102
Homework #9
Due date: April 5, 2007

<u>Group work on h/w assignments is not allowed. No credit is given for results without a solution or an explanation. Late homework is not accepted.</u>

Sections 4.3, 4.4

Problem I. Consider the matrix $A=\begin{bmatrix} 0 & 2 & 1 & 1 \\ 1 & 0 & 0 & 3 \\ 2 & 4 & 0 & 1 \\ 5 & 0 & 0 & 2 \end{bmatrix}$.

a) Use the "big formula" to find det A. Draw the paths connecting non-zero elements, and indicate clearly how you determine the sign for each path (i.e. count the number of permutations for each path).
b) Repeat the calculation using cofactors. Use a row or a column with the fewest elements.

Problem II. Do the following calculations for a non-singular matrix $A=\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 3 \\ 3 & 2 & 1 \end{bmatrix}$:

a) Find the cofactor matrix C
b) Use one of the rows or columns of C to find the determinant of this matrix. Check your calculation using a different row or column.
c) Find the inverse of the matrix $A^{-1} = C^T/(\det A)$, and check your answer by verifying that $AA^{-1}=I$

Problem III. Use the Cramer's Rule to solve $\begin{cases} 3x + y - z = 1 \\ x + 2y + z = 0 \\ 2x + y = 0 \end{cases}$

Problem IV.

a) A triangle has vertices from (0,0) to (2,1) and to (1,3). Find the surface area of this triangle.
b) A parallelepiped has edges from (1,1,1) to (4,2,2), (2,4,2) and (2,2,4). Find the volume of this parallelepiped. (Hint: first, find the edge vectors forming the sides of this box by subtracting the coordinates of the first vertex).

Problem V. Find the pivots of the matrix below. Instead of elimination, use the recursive formula $d_k = \dfrac{\det A_k}{\det A_{k-1}}$, where A_k is the $k \times k$ top-left block (submatrix) of A, and det A_0 = 1 by definition.

$$A=\begin{bmatrix} 2 & 1 & 3 \\ 2 & 3 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$